1 Reviewing number concepts

1.1 Different types of numbers

- Real numbers can be divided into rational and irrational numbers. You will deal with rational numbers in this chapter. Irrational numbers are covered in chapter 9.
- Rational numbers can be written as fractions in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
  (Integers are negative and positive whole numbers, and zero.)
- Integers, fractions and terminating decimals are all rational numbers.

Tip
Make sure you know what the following sets of numbers are: natural numbers, integers, odd and even numbers and prime numbers.

Exercise 1.1

1 Copy and complete this table by writing a definition and giving an example of each type of number.

Mathematical name	Definition	Example
Natural numbers		
Integers		
Prime numbers		
Square numbers		
Fraction		
Decimal		

2 Include numbers to show what each of the following symbols means. For example $100 > 99$.

(a) $>$ **(b)** $\leq$ **(c)** $\approx$ **(d)** $\therefore$
(e) $\sqrt{\ }$ **(f)** $\neq$ **(g)** $\geq$ **(h)** $<$

3 Look at this set of numbers.

$3, -2, 0, 1, 9, 15, 4, 5, -7, 10, 32, -32, 21, 23, 25, 27, 29, \frac{1}{2}$

(a) Which of these numbers are NOT natural numbers?
(b) Which of these numbers are NOT integers?
(c) Which of these numbers are prime numbers?
(d) Which of these numbers are square numbers?

4 List:

(a) four square numbers greater than 100.
(b) four rational numbers smaller than $\frac{1}{3}$.
(c) two prime numbers that are > 80.
(d) the prime numbers < 10.

1.2 Multiples and factors

- A multiple of a number is the product obtained when multiplying that number and an integer. The lowest common multiple (LCM) of two or more numbers is the lowest number that is a multiple of both (or all) of the numbers.
- A factor of a number is any number that will divide into the number exactly.
- The highest common factor (HCF) of two or more numbers is the highest number that is a factor of all the given numbers.

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

FAST FORWARD ▶

You will use LCM again when you work with fractions to find the lowest common denominator of two or more fractions. See chapter 5. ▶

Exercise 1.2 A

1 Find the LCM of the given numbers.

(a) 9 and 18 **(b)** 12 and 18 **(c)** 15 and 18 **(d)** 24 and 12
(e) 36 and 9 **(f)** 4, 12, and 8 **(g)** 3, 9 and 24 **(h)** 12, 16 and 32

2 Find the HCF of the given numbers.

(a) 12 and 18 **(b)** 18 and 36 **(c)** 27 and 90 **(d)** 12 and 15
(e) 20 and 30 **(f)** 19 and 45 **(g)** 60 and 72 **(h)** 250 and 900

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

Exercise 1.2 B

1 Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 me on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?

2 In a shopping mall promotion every 30th shopper gets a $10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives a voucher and a free meal?

3 Amanda has 40 pieces of fruit and 100 sweets to share amongst the students in her class. S is able to give each student an equal number of pieces of fruit and an equal number of swe What is the largest possible number of students in her class?

4 Sam has sheets of green and yellow plastic that he wants to use to make a square chequerboard pattern on a coffee table top. Each sheet measures 210 cm by 154 cm. The squares are to be the maximum size possible. What will be the length of the side of each square and how many will he be able to cut from each sheet?

1.3 Prime numbers

- Prime numbers only have two factors: 1 and the number itself.
- Prime factors are factors of a number that are also prime numbers.
- You can write any number as a product of prime factors. But remember the number 1 itself is *not* a prime number so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two or more numbers.

You can use a tree diagram or division to find the prime factors of a composite whole number.

Exercise 1.3

1 Identify the prime numbers in each set.

(a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
(b) 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
(c) 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

(a) 36 **(b)** 65 **(c)** 64 **(d)** 84
(e) 80 **(f)** 1000 **(g)** 1270 **(h)** 1963

3 Find the LCM and the HCF of the following numbers by means of prime factors.

(a) 27 and 14 **(b)** 85 and 15 **(c)** 96 and 27 **(d)** 53 and 16
(e) 674 and 72 **(f)** 234 and 66 **(g)** 550 and 128 **(h)** 315 and 275

1.4 Powers and roots

- A number is squared (n^2) when it is multiplied by itself ($n \times n$).
- The square root ($\sqrt{n}$) of a number is the number that is multiplied by itself to get the number.
- A number is cubed (n^3) when it is multiplied by itself and then multiplied by itself again ($n \times n \times n$).
- The cube root ($\sqrt[3]{n}$) of a number is the number that is multiplied by itself twice to get the number.

FAST FORWARD ▶

Powers greater than 3 are dealt with in chapter 2. See topic 2.5 indices. ▶

Exercise 1.4

1 Calculate.

(a) 3^2 **(b)** 18^2 **(c)** 21^2 **(d)** 25^2
(e) 6^3 **(f)** 15^3 **(g)** 18^3 **(h)** 35^3

2 Find these roots.

(a) $\sqrt{121}$ **(b)** $\sqrt[3]{512}$ **(c)** $\sqrt{441}$
(d) $\sqrt[3]{1331}$ **(e)** $\sqrt[3]{46656}$ **(f)** $\sqrt{2601}$
(g) $\sqrt{3136}$ **(h)** $\sqrt{729}$

If you don't have a calculator, you can use the product of prime factors to find the square root or cube root of a number.

3 Find all the square and cube numbers between 100 and 300.

4 Which of the following are square numbers and which are cube numbers?

1, 24, 49, 64, 256, 676, 625, 128

5 Simplify.

(a) $\sqrt{9}+\sqrt{16}$ **(b)** $\sqrt{9+16}$ **(c)** $\sqrt{64}+\sqrt{36}$
(d) $\sqrt{64+36}$ **(e)** $\sqrt{\frac{36}{4}}$ **(f)** $\left(\sqrt{25}\right)^2$
(g) $\frac{\sqrt{9}}{\sqrt{16}}$ **(h)** $\sqrt{169-144}$ **(i)** $\sqrt[3]{27}-\sqrt[3]{1}$
(j) $\sqrt{100 \div 4}$ **(k)** $\sqrt{1}+\sqrt{\frac{9}{16}}$ **(l)** $\sqrt{16} \times \sqrt[3]{27}$

1.5 Working with directed numbers

- Integers are directed whole numbers.
- Negative integers are written with a minus (–) sign. Positive integers may be written with a plus (+) sign, but usually they are not.
- In real life, negative numbers are used to represent temperatures below zero; movements downwards or left; depths distances below sea level; bank withdrawals and overdrawn amounts, and many more things.

Draw a number line to help you.

Exercise 1.5

1 If the temperature is 4 °C in the evening and it drops 7 °C overnight, what will the temperature be in the morning?

2 Which is colder in each pair of temperatures?

(a) 0 °C or –2 °C **(b)** 9 °C or –9 °C **(c)** –4 °C or –12 °C

3 An office block has three basement levels (level –1, –2 and –3), a ground floor and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?

(a) Starts on ground and goes down one floor then up five?
(b) Starts on level –3 and goes up 10 floors?
(c) Starts on floor 12 and goes down 13 floors?
(d) Starts on floor 15 and goes down 17 floors?
(e) Starts on level –2, goes up seven floors and then down eight?

1.6 Order of operations

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- The word 'of' means × and a fraction line means divide.
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.

Remember the order of operations using BODMAS:
*B*rackets
*O*f
*D*ivide
*M*ultiply
*A*dd
*S*ubtract

FAST FORWARD ▶

The next section will remind you of the rules for rounding numbers. ▶

Exercise 1.6

1 Calculate and give your answer correct to two decimal places.

(a) $8+3\times6$ **(b)** $(8+3)\times6$ **(c)** $8\times3-4\div5$

(d) $12.64+2.32\times1.3$ **(e)** $6.5\times1.3-5.06$ **(f)** $(6.7\div8)+1.6$

(g) $1.453+\frac{7.6}{3.2}$ **(h)** $\frac{5.34+3.315}{4.03}$ **(i)** $\frac{6.54}{2.3}-1.08$

(j) $\frac{5.27}{1.4\times1.35}$ **(k)** $\frac{11.5}{2.9-1.43}$ **(l)** $\frac{0.23\times4.26}{1.32+3.43}$

(m) $8.9-\frac{8.9}{10.4}$ **(n)** $\frac{12.6}{8.3}-\frac{1.98}{4.62}$ **(o)** $12.9-2.03^2$

(p) $(9.4-2.67)^3$ **(q)** $12.02^2-7.05^2$ **(r)** $\left(\frac{16.8}{9.3}-1.01\right)^2$

(s) $\dfrac{4.07^2}{8.2-4.09}$ (t) $6.8+\dfrac{1.4}{6.9}-\dfrac{1.2}{9.3}$ (u) $4.3+\left(1.2+\dfrac{1.6}{5}\right)^2$

(v) $\dfrac{6.1}{2.8}+\left(\dfrac{2.1}{1.6}\right)^2$ (w) $6.4-(1.2^2+1.9^2)^2$ (x) $\left(4.8-\dfrac{1}{9.6}\right)\times 4.3$

1.7 Rounding numbers

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
  - look at the value of the digit to the right of the place you are rounding to
  - if this value is ≥ 5 then you round up (add 1 to the digit you are rounding to)
  - if this value is ≤ 4 then leave the digit you are rounding to as it is.
- To round to a significant figure:
  - the first non-zero digit (before or after the decimal place in a number) is the first significant figure
  - find the correct digit and then round off from that digit using the rules above.

Exercise 1.7

FAST FORWARD ▶

Rounding is very useful when you have to estimate an answer. You will deal with this in more detail in chapter 5. ▶

1 Round these numbers to:

(i) two decimal places
(ii) one decimal place
(iii) the nearest whole number.

(a) 5.6543 **(b)** 9.8774 **(c)** 12.8706
(d) 0.0098 **(e)** 10.099 **(f)** 45.439
(g) 13.999 **(h)** 26.001

2 Round each of these numbers to three significant figures.

(a) 53217 **(b)** 712984 **(c)** 17.364 **(d)** 0.007279

3 Round the following numbers to two significant figures.

(a) 35.8 **(b)** 5.234 **(c)** 12345 **(d)** 0.00875
(e) 432128 **(f)** 120.09 **(g)** 0.00456 **(h)** 10.002

Mixed exercise

1 List the integers in the following set of numbers.

$\frac{3}{4}$ 24 0.65 −12 $3\frac{1}{2}$ 0 −15 0.66

2 List the first five multiples of 15.

3 Find the lowest common multiple of 12 and 15.

4 Write each number as a product of its prime factors.

(a) 196 **(b)** 1845 **(c)** 8820

5 Find the HCF of 28 and 42.

6 Simplify:

(a) $\sqrt{100} \div \sqrt{4}$ **(b)** $\sqrt{100 \div 4}$ **(c)** $\left(\sqrt[3]{64}\right)^3$ **(d)** $4^3 + 9^2$

7 Calculate. Give your answer correct to two decimal places.

(a) $\frac{5.4 \times 12.2}{4.1}$ **(b)** $\frac{12.2^2}{3.9^2}$ **(c)** $\frac{12.65}{2.04} + 1.7 \times 4.3$

(d) $\frac{3.8 \times 12.6}{4.35}$ **(e)** $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ **(f)** $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2 (3.3)^2$

8 Round each number to three significant figures.

(a) 1235.6 **(b)** 0.76513 **(c)** 0.0237548 **(d)** 31.4596

> **Tip**
> Most modern scientific calculators apply the rules for order of operations automatically. But if there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.